Nucleon pairing energy in even-even nuclei

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and a little bit more

Motivation

- How the pair interaction of nucleons in nuclei differs from that of free nucleons, at least for valence nucleons, remains an open question
- Do dineutron and diproton clustering exist in atomic nuclei?

Approach criterion

The proximity of experimental and calculated nucleon pairing energies.

Pairing energy E_{nn} *and* E_{pp} Definition:

$$E_{nn} = E_{b,N+2} - E_{b,N+1};$$
$$E_{pp} = E_{b,Z+2} - E_{b,Z+1};$$

$$E_{b,N} = c^2\Big(M(Z,N-1) + m_n - M(Z,N)\Big);$$
$$E_{b,Z} = c^2\left(M(Z-1,N) + m_p - M(Z,N)\right).$$

Experimental E_{nn} and E_{pp}

- **Experimental data** from

"Nuclear Wallet Cards database version of 5/16/2019" https://www.nndc.bnl.gov/nudat3/indx_sigma.jsp

- **Example**

Pairing energy for ${}^{10}_{6}C$ comprises **7.02 MeV**, while the pairing energy for ${}^{224}_{92}U$ is 3.8 times less and is equal to **1.83 MeV**. If you use energy of a nuclear quantum $\boldsymbol{\hbar\omega = \frac{41}{A^{\frac{1}{3}}}\ MeV}$,

then $\boldsymbol{E_{nn}/\hbar\omega}$ for ${}^{10}_{6}C$ is **0.371**, and that for ${}^{224}_{92}U$ is **0.270**

Experimental E_{nn} *and* E_{pp}

$E_{nn}/\hbar\omega \rightarrow$ average $E_{nn}/\hbar\omega$ over known isotopes

$E_{pp}/\hbar\omega \rightarrow$ average $E_{pp}/\hbar\omega$ over known isotones

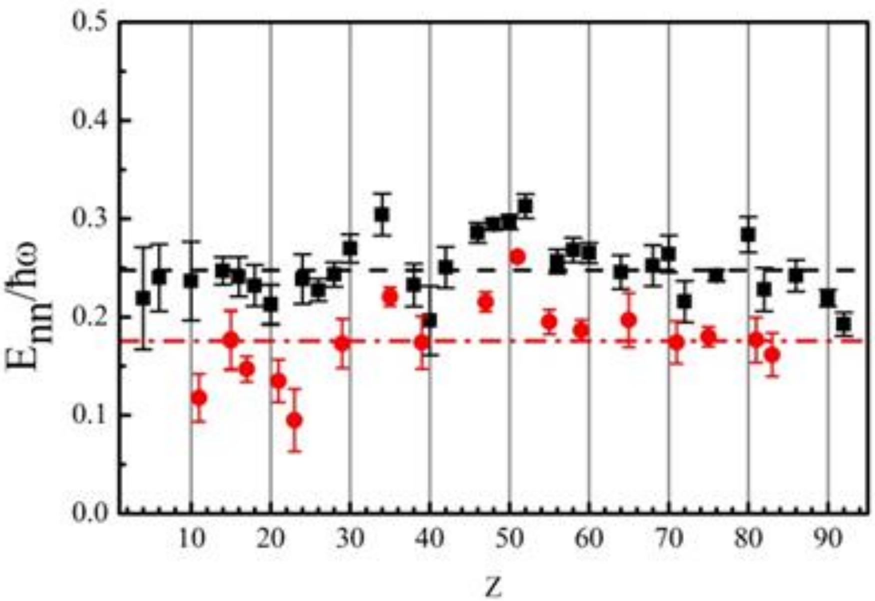


Fig.1. Experimental energies of *nn*-pairing. Filled squares – even *Z*, filled circles – odd *Z*.

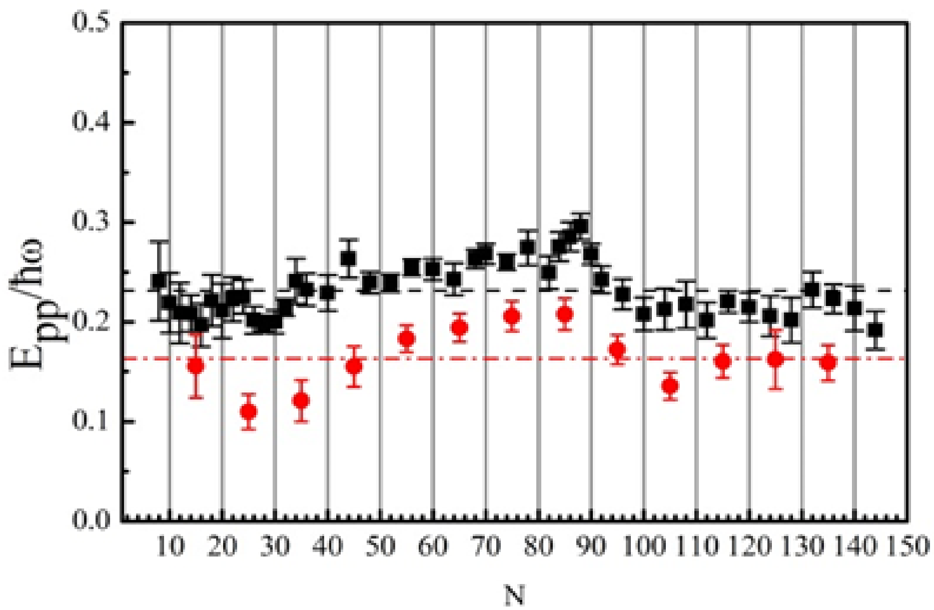


Fig.2. Experimental *pp*-pairing energies. Filled squares – even *N*, filled circles – odd *N*.

Experimental E_{nn} *and* E_{pp}

Nuclei	$\bar{E}_{nn}/\hbar\omega$	$\bar{E}_{pp}/\hbar\omega$
even	0.248 ± 0.005	0.232 ± 0.004
odd	0.176 ± 0.009	0.163 ± 0.008

Model

Hamiltonian H_1 for a nucleon with mass m

$$H_1 = -\frac{1}{2m}\Delta + \frac{m\omega^2}{2}r^2, \qquad E_1 = \omega\left(\frac{3}{2} + l + 2n\right):$$

$$n = 0,1,2,\dots$$

Two non-interacting nucleons must have the energy

$$E_2 = \omega(3 + 2l + 4n), \qquad n = 0,1,2,\dots \quad .$$

The Hamiltonian of two non-interacting particles with coordinates $\boldsymbol{r_1}$ and $\boldsymbol{r_2}$ in an oscillator well can be written in terms of the coordinates of the center of inertia of the pair $\boldsymbol{R} = \frac{1}{2}(\boldsymbol{r_1} + \boldsymbol{r_2})$ and the coordinates of the relative motion $\boldsymbol{r} = \boldsymbol{r_2} - \boldsymbol{r_1}$ in the form:

Model

$$H_2^0 = -\frac{1}{m}\Delta_{\boldsymbol{r}} - \frac{1}{4m}\Delta_{\boldsymbol{R}} + \frac{m\omega^2 r^2}{4} + m\omega^2 \boldsymbol{R}^2 = H_r^0 + H_R^0;$$

$$H_r^0 = -\frac{1}{m}\Delta_{\boldsymbol{r}} + \frac{m\omega^2 r^2}{4}; \;\; E_r^0 = \omega\left(\frac{3}{2} + l_r + 2n_r\right);$$

$$H_R^0 = -\frac{1}{4m}\Delta_{\boldsymbol{R}} + m\omega^2 \boldsymbol{R}^2; \;\; E_R^0 = \omega\left(\frac{3}{2} + l_R + 2n_R\right).$$

$$E_2 = 2E_1 = \omega(3 + 2l + 4n) = E_r^0 + E_R^0 = \omega(3 + l_r + l_R + 2n_r + 2n_R),$$

$$l_r = 0; \; l_R = 0, n_R = 0 \quad => 2l + 4n = 2n_r =>$$

$$E_r^0 = \omega\left(\frac{3}{2} + 2(l + 2n)\right)!$$

The value of l+2n is determined by the quantum numbers of motion of ONE nucleon

Model

The inclusion of the S-wave pair interaction allows us to write the radial Schrödinger equation for relative motion in the form:

$$-\frac{1}{m}\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}\Psi+\frac{m\omega^2 r^2}{4}\Psi+V\Psi=E_r^{in}\Psi\,.$$

V is the potential of nucleon-nucleon interaction

$$E_{nn}=E_r^0-E_r^{in}.$$

NN-potential

Yamaguchi's separable potential was chosen as a realistic nucleon-nucleon interaction model. In momentum representation, this potential takes the form

$$V(p,p') = \nu(p)\nu(p') = -\frac{8\pi}{m}\frac{\beta(\beta+\kappa)^2}{(\beta^2+p^2)(\beta^2+p'^2)}$$

Yamaguchi potential in the configuration space also has a separable form

$$V(r,r') = \nu(r)\nu(r') = -\frac{\beta(\beta+\kappa)^2}{2\pi m}\frac{e^{-\beta r}}{r}\frac{e^{-\beta r'}}{r'}$$

NN-potential

For separable potentials

$$\hat{V}\Psi = \nu(r)\int \nu(r')\Psi(r')d^3r' = \nu(r)C$$

Scattering parameters with the Yamaguchi potential give

$$a_2 = \frac{2(\beta+\kappa)^2}{\kappa(2\beta+\kappa)\beta}, \qquad r_{eff} = \frac{(\beta+\kappa)^2 + 2\beta^2}{\beta(\beta+\kappa)^2}.$$

Below, we use the accepted average values

	a_2 (fm)	r_{eff} (fm)
$\mathrm{T}=1, \mathrm{S}=0$	-18.5 ± 0.3	2.75 ± 0.11
$\mathrm{T}=0, \mathrm{S}=1$	5.424 ± 0.003	1.760 ± 0.005

Equation solutions

Let's introduce regular F and irregular G solutions for the harmonic oscillator equation with energy E and oscillatory length $x_0^2 = \frac{2}{m\omega}$:

$$F = \quad e^{-\frac{r^2}{2x_0^2}} r M\left(\frac{3\omega - 2E}{4\omega}, \frac{3}{2}, \frac{r^2}{x_0^2}\right),$$

$$G = \quad e^{-\frac{r^2}{2x_0^2}} r U\left(\frac{3\omega - 2E}{4\omega}, \frac{3}{2}, \frac{r^2}{x_0^2}\right),$$

in terms of Kummer's functions $M(a, b, z)$ and function $U(a,b,z)$

Equation solutions

We write down the solution of Eq. in the form:

$$r\Psi \propto \frac{C}{W}\left[\int_0^r dr'\, e^{-\beta r'}\, G(r)F(r') + \int_r^\infty dr' e^{-\beta r'}\, F(r)G(r')\right].$$

In spectral points, at $E = E_r^{in}$, defined by the equation:

$$\mathbf{1} = \frac{\mathbf{4}\boldsymbol{\beta}(\boldsymbol{\beta}+\boldsymbol{\kappa})^{\mathbf{2}}}{\boldsymbol{W}}\int_{\mathbf{0}}^{\infty} \boldsymbol{dr}\, \boldsymbol{e}^{-\boldsymbol{\beta r}}\boldsymbol{G}(\boldsymbol{r})\int_{\mathbf{0}}^{\boldsymbol{r}} \boldsymbol{dr'}\, \boldsymbol{e}^{-\boldsymbol{\beta r'}}\boldsymbol{F}(\boldsymbol{r'})$$

Here $W = F'G - G'F$.

Remarks

After repeated integration by parts, we obtain a simplified form of the spectral equation

$$\frac{G'(0)}{G(0)} = -\frac{\kappa(2\beta+\kappa)\beta}{2(\beta+\kappa)^2} + \frac{3}{2\,\beta}K^2 + O\big((\beta x_0)^{-2}\big);$$

$$K^2 = mE.$$

And at $\beta x_0 \to \infty$

$$\frac{\Gamma\left(\frac{3}{4} - \frac{E_r^{in}}{2\omega}\right)}{\Gamma\left(\frac{1}{4} - \frac{E_r^{in}}{2\omega}\right)}\frac{2}{x_0} = \frac{1}{a_2}$$

Results

Zero range interactions $(\beta x_0 \to \infty)$

$$E_{nn}/\omega \approx 1$$
$$E_{pn}/\omega \approx 1$$

Zero range interactions does not describe the nucleon pairing energy in the model under consideration

Results
Potential of Yamaguchi

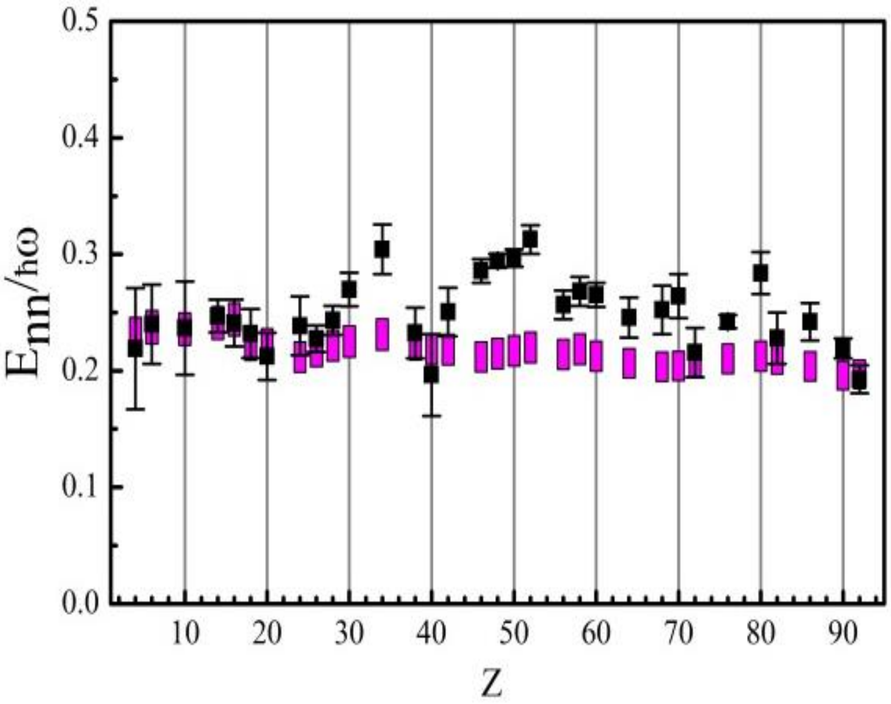


Fig. 3. Experimental and calculated energies of *nn*-pairing

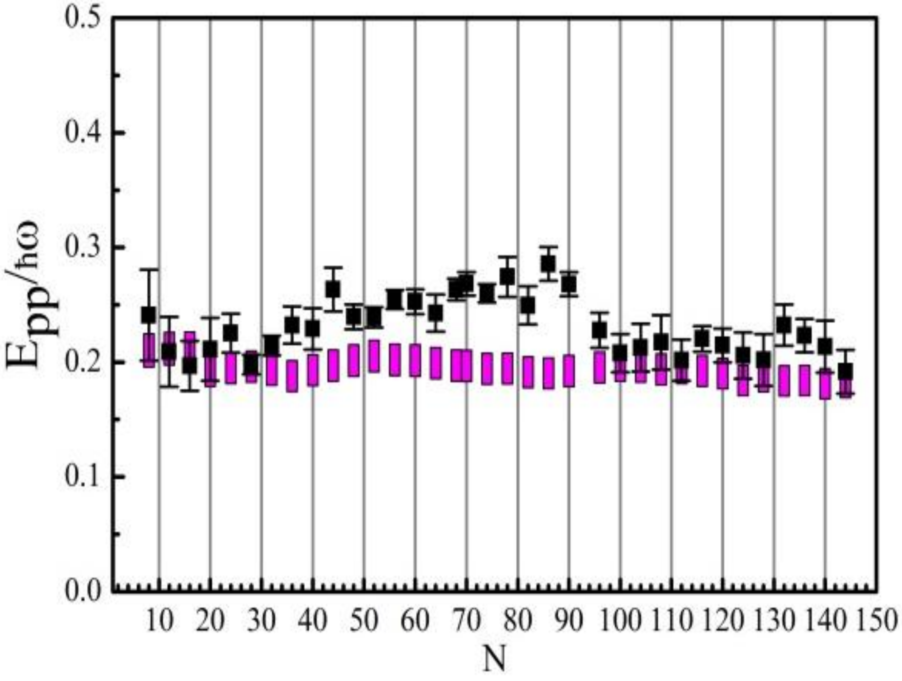


Fig. 4. Experimental and calculated energies of *pp*-pairing

Results
Potential of Yamaguchi

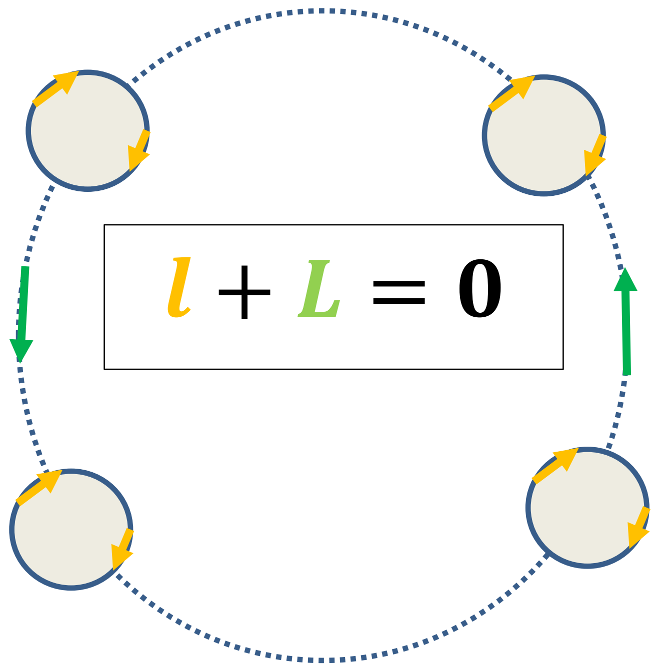
Nucleon pairing energy	Experiment	Calculation	Calculation without the Coulomb repulsion
$\bar{E}_{nn}/\omega$	0.248 ± 0.005	0.217 ± 0.013	
$\bar{E}_{pp}/\omega$	0.232 ± 0.004	0.195 ± 0.013	0.234 ± 0.013

Discussion. Pairing scheme

Nobel laureate Maria Goeppert Mayer – «Think of a room full of waltzers. … all the dancers are spinning twirling round and round like tops as they circle the room, each pair both twirling and circling».

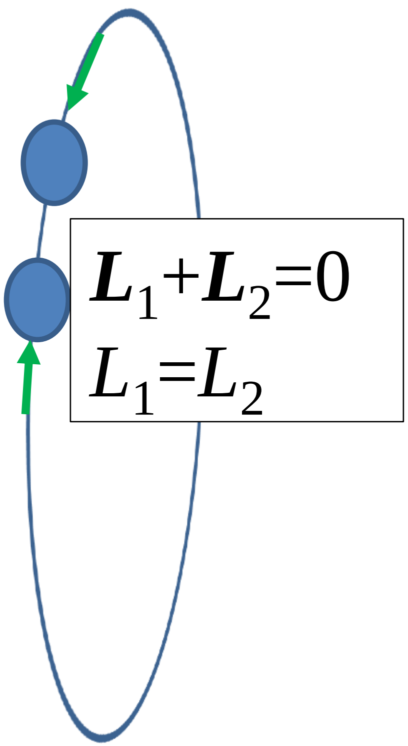
Discussion. Pairing scheme

A simplified view of MGM pairing



Discussion. Pairing scheme

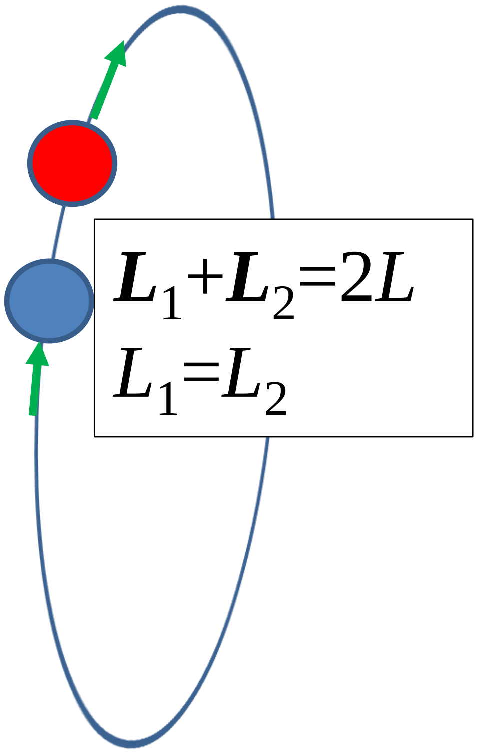
Pairings for T=1 (*nn*, *pp*). Consequence of the present model.



There is no clustering of pairs. In fact, the mean square distance between nucleons in an interacting pair differs from that in a pair without interaction by less than 1%.

Discussion. Pairing scheme

Pairings for T=0 (*np*). Consequence of the present model.



There should be clustering. But from the chain of equalities

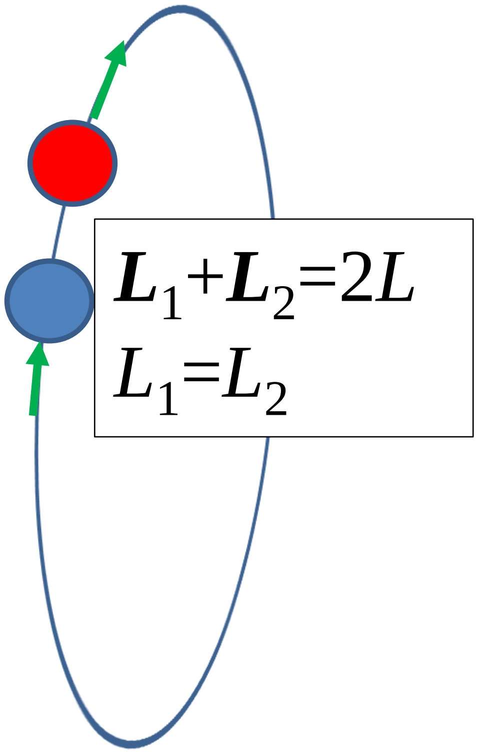
$$2l + 2n_r + 2n_R = 2l + 4n$$
$$n_r + n_R = 2n$$

follows that

$$E_r^0 = \omega(3/2 + 2(2n - n_R)),$$
$$n_R = ???$$

Discussion. Pairing scheme

Pairings for T=0 (*np*). Consequence of the present model.



For example, for nuclei 10B, 14N and 18F, with experimental energies of *pn*-pairing:

0.356, 0.330, 0.320,

calculations with n_R=1 give

0.364, 0.336, 0.305.

Similar, but n_R looks like a fitting parameter

Conclusions

The description of the pairing energy in the presented model is quite adequate.

But the next step requires solving the problem of pairing nucleons in a potential that does not allow the separation of the pair's center of mass.

For example, in the Woods–Saxon potential.

The problem becomes four-dimensional!

Thank you for attention