

Scattering in Three-body Coulomb System

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Plan of the talk

- I. Scattering processes in three-body system
- II. Effective interaction in two-body sector of the three-body system with Coulomb interaction. CCE approach
- III. Dipole interaction in CCE
- IV. Model free formalism for Three Body Coulomb Scattering Problem
- V. *Ab initio* calculation of the scattering problem in $e^-e^-p^+$, $e^-e^+p^-$ and e^-He^+ systems
- IV. Conclusion



I. Scattering Processes in Three-Body System

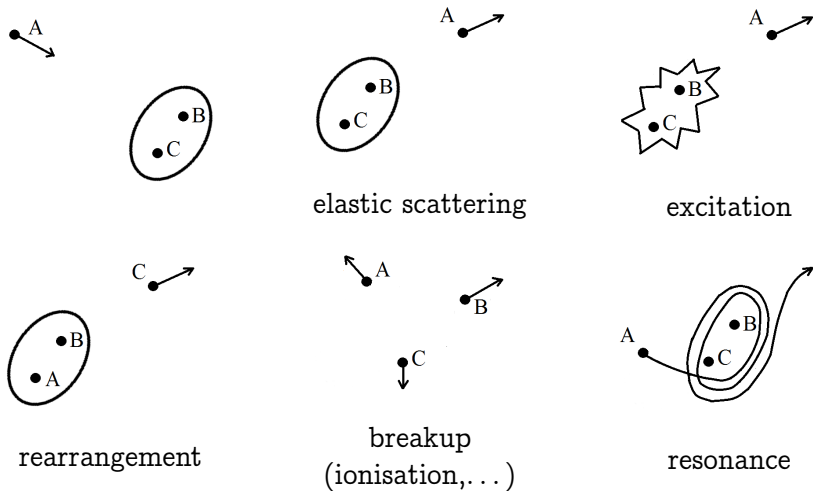


Figure: Scattering processes in a three-body system.



I. Scattering Processes in Three-Body System

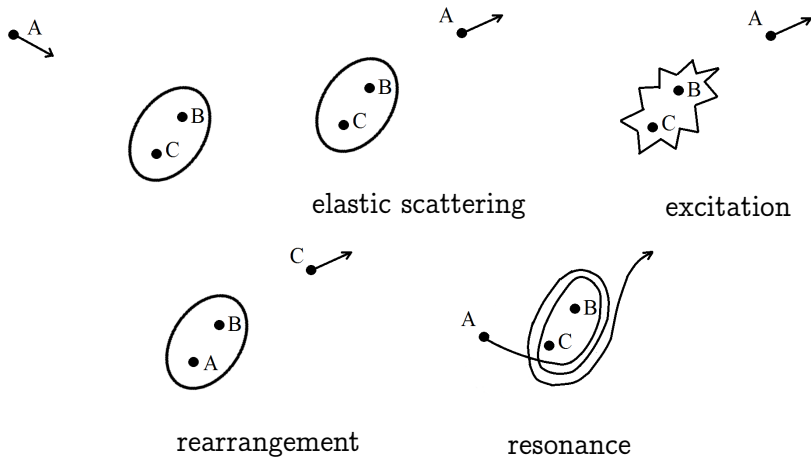


Figure: **Binary** scattering processes in a three-body system below breakup threshold



Jacobi vectors for three-body system

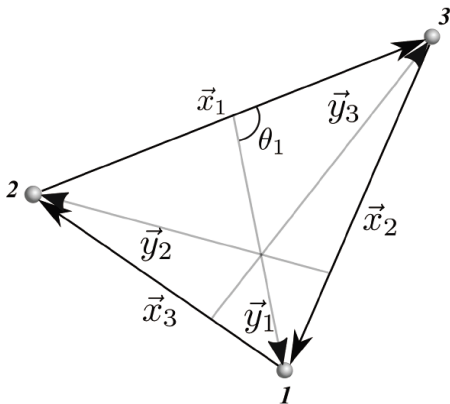


Figure: Jacobi coordinates



II. Effective Interaction in Two-Body Sector of the Three-Body System with Coulomb Interaction

The 3-body Hamiltonian in the center of mass frame:

$$H = H_0 + V \equiv -\Delta_{\mathbf{x}_1} - \Delta_{\mathbf{y}_1} + \sum_{a=1}^3 V_a(\mathbf{x}_a),$$

$\{\mathbf{x}_a, \mathbf{y}_a\}$ is the set of standard mass-weighted Jacobi coordinates:

$$\mathbf{x}_a = \sqrt{\mu_{bc}} \mathbf{x}_a, \quad \mathbf{y}_a = \sqrt{\mu_{a,bc}} \mathbf{y}_a.$$

$V_a(\mathbf{x})$ are two body potentials:

$$V_a(\mathbf{x}) = \frac{q_b q_c}{|\mathbf{x}|}$$

$\{a, b, c\}$ runs over $\{1, 2, 3\}$ cyclically. $\hbar = 1$ throughout.



II.1. Two-body sectors

$$|\mathbf{x}_1| \ll |\mathbf{y}_1|$$

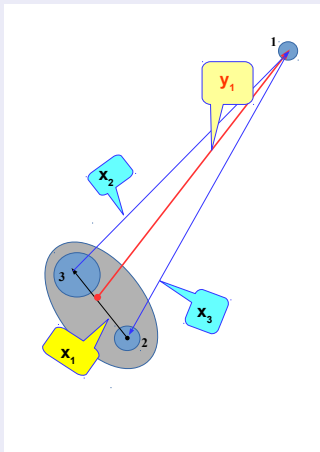


Figure: The configuration of bound state of particles (2,3) as a target and particle 1 as a spectator

II.2. Interactions of particle 1 with particles 2 and 3 in the two-body sector $|\mathbf{x}_1| \ll |\mathbf{y}_1|$

Multipole expansion of Coulomb interactions:

$$\begin{aligned} \sum_{a=2}^3 \frac{q_1 q_a}{|\mathbf{x}_a|} &= \sum_{a=2}^3 \frac{q_1 q_a}{|c_{a1} \mathbf{x}_1 + s_{a1} \mathbf{y}_1|} = \\ &= \sum_{a=2}^3 q_1 q_a \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} (-1)^{\ell} \frac{4\pi}{2\ell+1} \frac{(|c_{a1} \mathbf{x}_1|)^{\ell}}{(|s_{a1} \mathbf{y}_1|)^{\ell+1}} Y_{\ell m}(\hat{\mathbf{x}}_1) Y_{\ell m}^*(\hat{\mathbf{y}}_1) = \\ &= \frac{1}{|\mathbf{y}_1|} \sum_{a=2}^3 \frac{q_1 q_a}{|s_{a1}|} - \frac{1}{|\mathbf{y}_1|^2} \sum_{a=2}^3 q_1 q_a \frac{|c_{a1} \mathbf{x}_1|}{|s_{a1}|} P_1(\hat{\mathbf{x}}_1 \cdot \hat{\mathbf{y}}_1) + O(|\mathbf{y}_1|^{-3}) \end{aligned}$$

Result:

$$V_2 + V_3 \sim \frac{C}{|\mathbf{y}_1|} + \frac{A(\mathbf{x}_1, \hat{\mathbf{y}}_1)}{|\mathbf{y}_1|^2} + O(|\mathbf{y}_1|^{-3}), \quad |\mathbf{y}_1| \rightarrow \infty$$

II.3. CCE approach to scattering of a charged particle 1 on a bound pair of charged particles 2,3

The typical approach is the close coupling expansion (CCE) (if rearrangement process are not taken into account) (Seaton, Burke, Gailitis...) within R-matrix formalism

CCE for wave function

$$\Psi(\mathbf{x}_1, \mathbf{y}_1) = \sum_{n\alpha} \frac{\Psi_{n\alpha}(\mathbf{y}_1)}{x_1 y_1} \phi_{nl}(x_1) \mathcal{Y}_\alpha(\hat{x}_1, \hat{y}_1), \quad \alpha = LM\ell\ell_1$$

where ϕ_{nl} is radial wave function of Coulomb bound state with the energy ϵ_n , \mathcal{Y}_α are bispherical harmonics corresponding to the total orbital momentum L .

Or

$$\Psi(\mathbf{x}_1, \mathbf{y}_1) = \sum_{n\alpha} \frac{\Psi_{n\alpha}(\mathbf{y}_1)}{x_1 y_1} |n\alpha\rangle$$



CCE representation of the Schrödinger equation

$$\langle n\alpha | [H_0 + V_1 + \sum_{b=2}^3 V_b - E] \sum_{n'\alpha'} |n'\alpha'\rangle \frac{\Psi_{n'\alpha'}}{x_1 y_1} = 0$$

Or

$$p_n^2 = E - \epsilon_n$$

$$\left(-\frac{d^2}{dy_1^2} + \frac{C}{y_1} + \frac{l_1(l_1 + 1)}{y_1^2} - p_n^2 \right) \Psi_{n\alpha} + \sum_{n'\alpha'} \langle n\alpha | \frac{A}{y_1^2} + \dots | n'\alpha' \rangle \Psi_{n'\alpha'} = 0 \quad (1)$$

Asymptotic matrix form when $y_1 \rightarrow \infty$

$$\left(-\frac{d^2}{dy_1^2} + \frac{C}{y_1} + \frac{l_1(l_1 + 1) + \mathbf{A}}{y_1^2} - \mathbf{p}^2 \right) \Phi(y_1) = O(y_1^{-3})$$

CCE equations for a long time were the main and **ONLY** tool for analyzing scattering of a charged particle on a two-body target bound by Coulomb potential ($e^- - \text{H}$, $e^- - \text{He}^+$, $e^+ - \text{H}$, ...)

Two main features of scattering of charged particles on two-body Coulomb target:

- Under threshold resonances
- Above threshold oscillations (GD = Gailitis, Damburg)

These features are derived from the solution of model CCE equations within the requirements that the dipole potential matrix \mathbf{A} has the same block structure than the matrix \mathbf{p}^2 , i.e.

$$\mathbf{p}^2 \sim p_n^2 \delta_{nn'} \delta_{\alpha\alpha'} \quad \mathbf{A} \propto \langle n\alpha | A | n\alpha' \rangle \delta_{nn'},$$

i.e. **by neglecting dipole coupling of target states with $n \neq n'$** .

III. Dipole interaction in CCE

Gailitis *Proc. Phys. Soc.*, 82:192–200, 1963

Yakovlev, Gradusov *Theor. Math. Phys.* 217:2 416-429, 2023

Model CCE equations for e^-H scattering:

Main consequence of diagonality of \mathbf{A} is $[\mathbf{A}, \mathbf{p}^2] = 0$ and hence the diagonalizing matrix \mathbf{W} such that $\mathbf{W}^\dagger [l_1(l_1 + 1) + \mathbf{A}]\mathbf{W} = \mathbf{D}$, $\mathbf{D}_{n\alpha, n'\alpha'} = d_{nl}\delta_{n\alpha, n'\alpha'}$ commutes with \mathbf{p}^2 , i.e. $[\mathbf{W}, \mathbf{p}^2] = 0$. This allows to diagonalize the CCE equations:

$$\left(-\frac{d^2}{dy_1^2} + \frac{C}{y_1} + \frac{\mathbf{D}}{y_1^2} - \mathbf{p}^2 \right) \mathbf{W}^\dagger \Phi(y_1) = 0$$

$$[\mathbf{W}^\dagger \Phi^\pm(y_1)]_{n\alpha} = H_{\mathcal{L}_{n\alpha}}^\pm(p_n y_1) \delta_{nn'} \delta_{\alpha\alpha'} \quad (2)$$

$$\mathbf{D} = \mathcal{L}(\mathcal{L} + 1), \quad \mathcal{L}_{n\alpha, n'\alpha'} = \mathcal{L}_{n\alpha} \delta_{n\alpha, n'\alpha'},$$

$$\mathcal{L}_{n\alpha} = -1/2 \pm \sqrt{1/4 + d_{n\alpha}}$$



III.1 Two possibilities for D:

- 1 $d_{n\alpha} \geq 0$ then with $\mathcal{L}_{n\alpha} \geq 0$ two solutions are given by (2)
- 2 there is $n\alpha$ such that $d_{n\alpha} < 0$ then
 - ▶ if $|d_{n\alpha}| < 1/4$ the new momenta $\mathcal{L}_{n\alpha}$ are real, the solutions are given by (2). For this case the equation

$$\left(-\frac{d^2}{dy_1^2} + \frac{C}{y_1} + \frac{d_{n\alpha}}{y_1^2} - E + \epsilon_n \right) [\mathbf{W}^\dagger \Phi]_{n\alpha}(y_1) = 0 \quad (3)$$

supports finite number of bound states. For full equation (1) where the coupling between channels with different n are not zero these bound states transforms into resonances (called as Feshbach resonances).

- ▶ if $|d_{n\alpha}| > 1/4$ then new momenta are complex $\mathcal{L}_{n\alpha} = -1/2 \pm i\sqrt{|d_{n\alpha}| - 1/4}$, the equation (3) supports infinitely many bound states accumulating to the threshold ϵ_n from below. For full equation (1) where the coupling between channels with different n are not zero these bound states transforms into series of resonances accumulating to the thresholds (called as Feshbach resonances).



III. 2 Asymptotic boundary conditions for full CCE equations (1). Oscillation of cross section when

$$|d_{n\alpha}| > 1/4$$

Standard boundary conditions for full CCE equations have the form

$$\Psi_{n\alpha}(y_1) \sim H_{\ell_1}^-(\eta_n, p_n y_1) \delta_{nn'} \delta_{\alpha\alpha'} - H_{\ell_1}^+(\eta_n, p_n y_1) S_{nn', \alpha\alpha'} \quad (4)$$

where $H_{\ell}^{\pm} = G_{\ell} \pm iF_{\ell}$, $\eta_n = C/(2p_n)$ and $S_{nn', \alpha\alpha'}$ is S-matrix. Set of asymptotic solutions (2) provides more sophisticated asymptotic condition that compensates as $y_1 \rightarrow \infty$ the diagonal blocks of dipole coupling matrix:

$$\begin{aligned} \Psi_{n\alpha}(y_1) \sim & W_{n\alpha, \alpha'} \delta_{nn'} H_{\mathcal{L}_{n'\alpha'}}^-(\eta'_n, p'_n y_1) - \\ & - \sum_{\alpha''} W_{n\alpha, \alpha''} H_{\mathcal{L}_{n\alpha''}}^+(\eta_n, p_n y_1) \hat{S}_{nn', \alpha''\alpha'}. \end{aligned} \quad (5)$$

The original S-matrix S then is calculated from renormalized one \hat{S} by similarity transformation

$$S = W J \hat{S} J^{\dagger} W^{\dagger}.$$



Oscillation of cross section when $|d_{n\alpha}| > 1/4$

If $d_{n\alpha} < -1/4$ then $\mathcal{L}_{n,\alpha}$ is complex $\mathcal{L}_{n\alpha} = -1/2 \pm i\sqrt{|d_{n\alpha}| - 1/4}$ and the T matrix p-dependence ($iT = S - 1$)

$$T_{n\alpha, n_0\alpha_0} \sim p_n^{2\mathcal{L}_{n,\alpha}+1} = \exp\{i2\Im(\mathcal{L}_{n\alpha}) \ln(p_n)\},$$

leads to the specific anomalous cross section oscillations

$$\sigma_{n\alpha, n_0\alpha_0} = A + B \cos[2\Im(\mathcal{L}_{n\alpha}) \ln(p_n) + \phi],$$

which gives rise to an infinite number of oscillations in the cross section as the energy approaches the threshold from above (GD oscillations formula 1963).



III.1. Importance of account for the dipole interaction in practical calculations

S. Yakovlev, V. Gradusov *Theor. Math. Phys.* 217:2 416-429, 2023

CCE model $\text{Ps}(n = 2) - \bar{p}$ scattering

$$\left[-\frac{d^2}{dy^2} - p^2 + \frac{1}{\rho^2(y)} \begin{pmatrix} 24.9947 & 0 \\ 0 & -22.9947 \end{pmatrix} \right] \Psi =$$

$$\left[-\frac{d^2}{dy^2} - p^2 + \frac{1}{\rho^2(y)} \Lambda(\Lambda + 1) \right] \Psi = 0$$

$$\Lambda = \text{diag}[4.52441, i4.76914], \Psi = [\Psi_1, \Psi_2]^T$$

$$\rho(y) = 6 \text{ a.u. if } y \leq 6 \text{ a.u.}, \rho(y) = y, \text{ if } y > 6 \text{ a.u.}$$



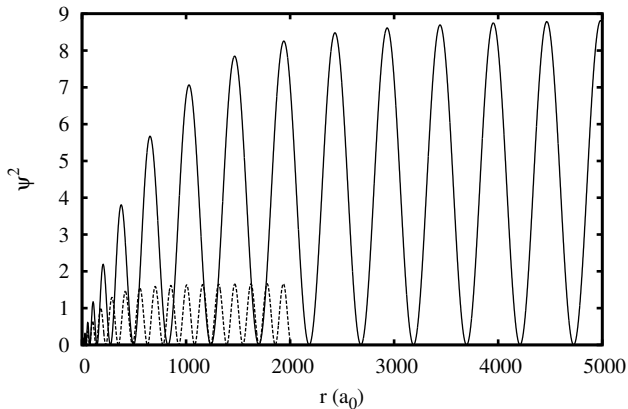


Figure: Squared Ψ_2 component of wave functions for different values of p : solid line corresponds to $p = 0.006 a_0^{-1}$, dashed line corresponds to $p = 0.02 a_0^{-1}$.



IV. Model free formalism for three-body Coulomb scattering problem

Merkuriev-Faddeev equations (MFE)

$$\begin{aligned} \left(-\Delta_{\mathbf{x}_1} - \Delta_{\mathbf{y}_1} + V_1(\mathbf{x}_1) + \sum_{b \neq 1} V_b^{(l)} - E \right) \psi_1(\mathbf{x}_1, \mathbf{y}_1) &= -V_1^{(s)} (\psi_2 + \psi_3), \\ \left(-\Delta_{\mathbf{x}_2} - \Delta_{\mathbf{y}_2} + V_2(\mathbf{x}_2) + \sum_{b \neq 2} V_b^{(l)} - E \right) \psi_2(\mathbf{x}_2, \mathbf{y}_2) &= -V_2^{(s)} (\psi_1 + \psi_3), \\ \left(-\Delta_{\mathbf{x}_3} - \Delta_{\mathbf{y}_3} + V_3(\mathbf{x}_3) + \sum_{b \neq 3} V_b^{(l)} - E \right) \psi_3(\mathbf{x}_3, \mathbf{y}_3) &= -V_3^{(s)} (\psi_1 + \psi_2). \end{aligned}$$

Splitting of Coulomb potential

$$V_a(\mathbf{x}) = V_a^{(s)}(\mathbf{x}) + V_a^{(l)}(\mathbf{x})$$

$$V_a^{(l)}(\mathbf{x}) = \frac{q_b q_c}{x} \theta_s(r - R), \quad \theta_s - \text{smoothed out theta function.}$$



Structure of the r.h.s. of MFE

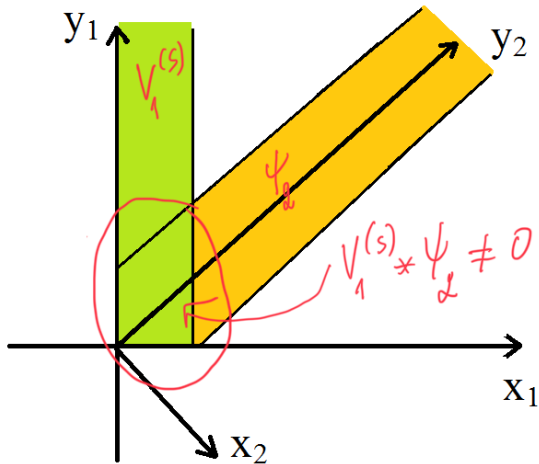


Figure: Green - $\text{supp}V_1^{(l)}$, Yellow - $\text{supp}\psi_2$



Asymptotic decoupling of MFE as $\{x_a, y_a\} \rightarrow \infty$

$$\left(-\Delta_{x_1} - \Delta_{y_1} + V_1(x_1) + \sum_{b \neq 1} V_b^{(l)} - E \right) \psi_1(x_1, y_1) = 0,$$

$$\left(-\Delta_{x_2} - \Delta_{y_2} + V_2(x_2) + \sum_{b \neq 2} V_b^{(l)} - E \right) \psi_2(x_2, y_2) = 0,$$

$$\left(-\Delta_{x_3} - \Delta_{y_3} + V_3(x_3) + \sum_{b \neq 3} V_b^{(l)} - E \right) \psi_3(x_3, y_3) = 0.$$

$$\psi_a(x_a, y_a) = \sum_{n\alpha} \frac{\psi_{a(n\alpha)}(y_a)}{x_a y_a} |n\alpha\rangle$$

Asymptotic equation for partial wave of MF component ψ_a

$$\left(-\frac{d^2}{dy_a^2} + \frac{\ell_1(\ell_1 + 1)}{y_a^2} - p_{an}^2 \right) \psi_{a(n\alpha)} + \sum_{n'\alpha'} \langle n\alpha | \sum_{b \neq a} V_b^{(l)} | n'\alpha' \rangle \psi_{a(n'\alpha')} = 0$$

Asymptotic boundary conditions for MF

Asymptotic waves which take into account full dipole coupling.
Gradusov Yakovlev Theor. Math. Phys. 2024

$$\phi_{a(n\alpha)(n'\alpha')}^{\pm}(y_a, p_{n'}) = \left[W_{(n\alpha)(n'\alpha')}^{a(0)} + \frac{1}{y_a^2} W_{(n\alpha)(n'\alpha')}^{a(1)} \right] H_{\mathcal{L}_a(n'\alpha')}^{\pm}(\eta_{n'}, p_{n'} y_a).$$

$$W_{(n\alpha)(n'\alpha')}^{a(0)} = \delta_{nn'} W_{n\alpha\alpha'}^a,$$

$$W_{(n\alpha)(n'\alpha')}^{a(1)} = (1 - \delta_{nn'}) \frac{\sum_{\alpha''} A_{(n\alpha)(n'\alpha'')}^a W_{n'\alpha''\alpha'}^a}{(p_n^2 - p_{n'}^2)},$$

$$\mathcal{L}_{a(n\alpha)}(\mathcal{L}_{a(n\alpha)} + 1) = q_{a(n\alpha)}$$

$q_{a(n\alpha)}$ and $W_{n\alpha\alpha'}^a$ are eigen values and eigen vectors of the block matrix

$$\ell_1(\ell_1 + 1)\delta_{\alpha\alpha'} + A_{(n\alpha)(n\alpha')}^a.$$



MF partial wave asymptotic for scattering problem

Standard boundary conditions for MF, incident channel $a(n'\alpha')$

$$\begin{aligned}\psi_{a(n\alpha)}(\mathbf{y}_a) &\sim H_{\ell_1}^-(\eta_{an}, \mathbf{p}_{an} \mathbf{y}_a) \delta_{nn'} \delta_{\alpha\alpha'} - H_{\ell_1}^+(\eta_{an}, \mathbf{p}_{an} \mathbf{y}_a) S_{a(n\alpha), a(n'\alpha')} \\ \psi_{b(n\alpha)}(\mathbf{y}_b) &\sim -H_{\ell_1}^+(\eta_{bn}, \mathbf{p}_{bn} \mathbf{y}_b) S_{b(n\alpha), a(n'\alpha')}\end{aligned}$$

Advanced boundary conditions, incident channel $a(n'\alpha')$

$$\begin{aligned}\psi_{a(n\alpha)}(\mathbf{y}_a) &\sim \phi_{a(n\alpha)(n'\alpha')}^-(\mathbf{y}_a, \mathbf{p}_{an'}) - \\ &\quad - \sum_{n''\alpha''} \phi_{a(n''\alpha'')(n'\alpha')}^+(\mathbf{y}_a, \mathbf{p}_{an'}) S(a(n''\alpha'')|a(n'\alpha')) \\ \psi_{b(n\alpha)}(\mathbf{y}_b) &\sim - \sum_{n''\alpha''} \phi_{b(n''\alpha'')(n'\alpha')}^+(\mathbf{y}_b, \mathbf{p}_{bn'}) S(b(n''\alpha'')|a(n'\alpha'))\end{aligned}$$



V. *Ab initio* calculation of the scattering problem in $e^-e^-p^+$, $e^-e^+p^-$ and e^-He^+ systems

- 3D tree-body MF equations in total orbital momentum representation
- Advanced asymptotic boundary conditions, which compensate long-range Coulomb and dipole interactions
- Quintic splines expansion of the solution
- Tensor trick preconditioning procedure for solving discretized MFE
- Complex rotation method for 3D Schrödinger equation



V.1. Calculation of e^- -H and e^- -He⁺ Scattering-Resonances $L = 0$

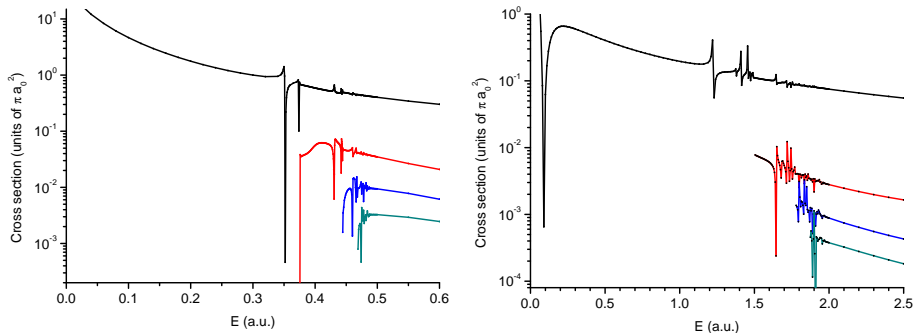


Figure: Left: The singlet $1s \rightarrow 1s, 2s, 3s, 4s$ cross sections for the e^- -H scattering as a function of the incident electron energy. The thresholds are 0, 0.375, 0.444 and 0.469 a.u., respectively. Right: The singlet $1s \rightarrow 1s, 2s, 3s, 4s$ cross sections for the e^- -He⁺ scattering as a function of the incident electron energy. The thresholds are 0, 1.5, 1.778 and 1.875 a.u., respectively.



V.2. $e^+e^-\bar{p}$ system

Gradusov, Roudnev, Yarevsky, Yakovlev J. Phys. B: At. Mol. Opt. Phys.

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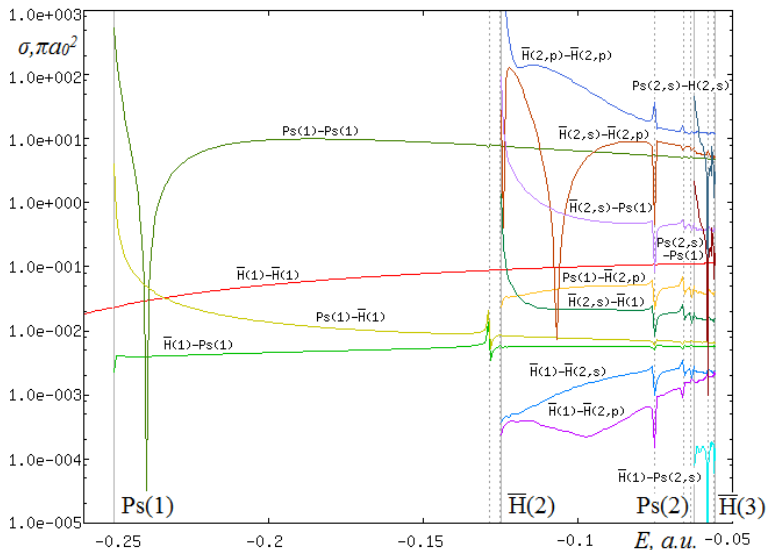
waves	$\bar{H}(n=1)$	Ps(n=1)	$\bar{H}(n=2)$ s, p	Ps(n=2) s, p	$\bar{H}(n=3)$ s, p, d
a.u.	-0.4997	-0.250	-0.1249	-0.0625	-0.0555
eV	-13.6	-6.80	-3.40	-1.70	-1.51

Table: Energy thresholds of $e^+e^-\bar{p}$ binary channels

6 open channels between Ps(n=2) and $\bar{H}(n=3)$ thresholds.



Cross sections $e^+e^-\bar{p}$



Ramsauer effect

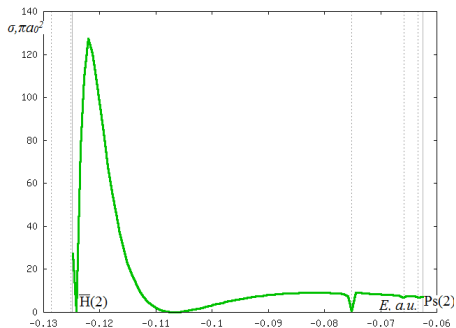
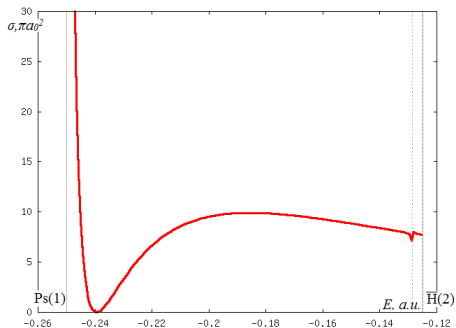


Figure: $\text{Ps}(1, s) \rightarrow \text{Ps}(1, s)$ cross section Figure: $\bar{\text{H}}(2, s) \rightarrow \bar{\text{H}}(2, p)$ cross section



V.3. Realistic Calculations of Multiconfiguration Collision in $e^+e^-\bar{p}$ system $L = 0$ via MFE

V. Gradusov, S. Yakovlev, JETP Letters 2024, 119:3, 151-157



GD oscillations in $\text{Ps} \rightarrow \text{Ps}$ collision

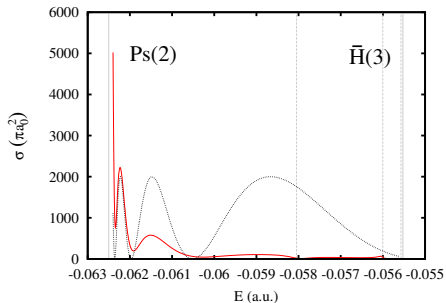


Figure: $\text{Ps}(2s) \rightarrow \text{Ps}(2s)$ cross section for $L = 0$

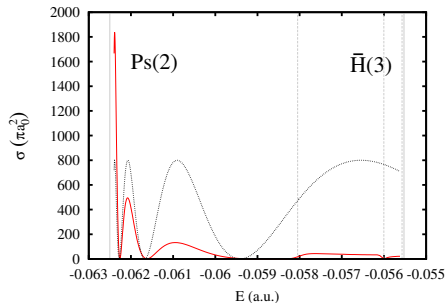


Figure: $\text{Ps}(2s) \rightarrow \text{Ps}(2p)$ cross section for $L = 0$



GD oscillations in $\text{Ps} \rightarrow \text{Ps}$ collision

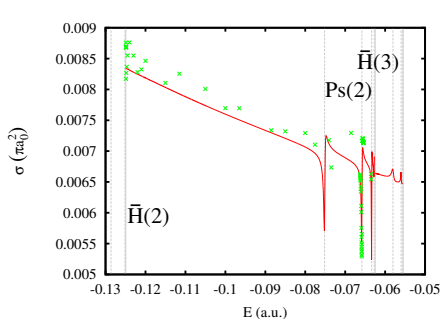


Figure: Antihydrogen formation cross sections $\text{Ps}(1) \rightarrow \bar{\text{H}}(1)$.

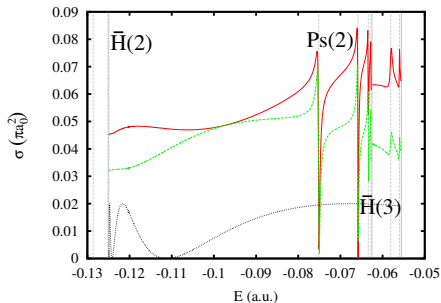


Figure: Antihydrogen formation cross sections $\text{Ps}(1) \rightarrow \bar{\text{H}}(2s)$ (solid) and $\text{Ps}(1) \rightarrow \bar{\text{H}}(2p)$ (dashed line)



G-D oscillations of $\text{Ps} \rightarrow \bar{\text{H}}$ production cross sections above $\bar{\text{H}}(3)$ threshold

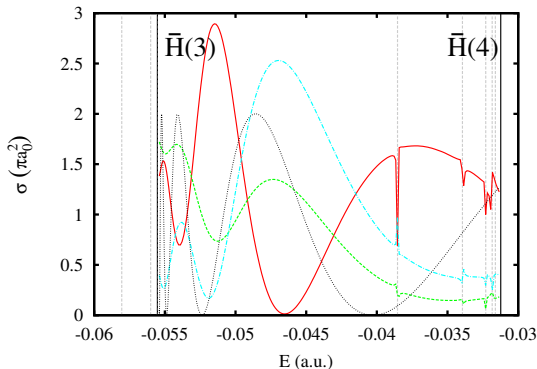


Figure: Fig. 5 Antihydrogen formation cross sections $\text{Ps}(2s) \rightarrow \bar{\text{H}}(3s)$ (red), $\text{Ps}(2s) \rightarrow \bar{\text{H}}(3p)$ (green) and $\text{Ps}(2s) \rightarrow \bar{\text{H}}(3d)$ (blue).



G-D oscillations in e^- -H collision

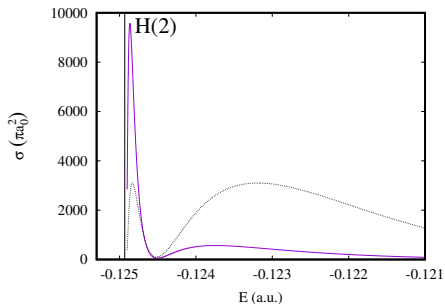


Figure: H(2s)-H(2s) cross section.

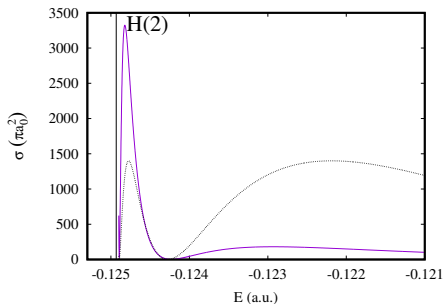


Figure: H(2s)-H(2p) cross section.



V.4. $e^+e^-He^{++}$ system.

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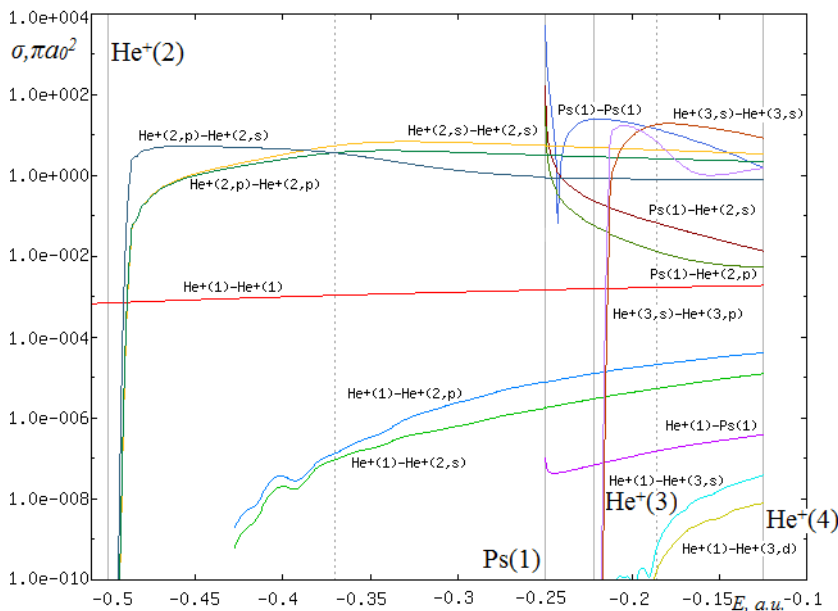
waves	$He^+(n=1)$	$He^+(n=2)$ s, p	Ps(n=1)	$He^+(n=3)$ s, p, d	$He^+(n=4)$ s, p, d, f
a.u.	-1.9997	-0.4999	-0.25	-0.2222	-0.1250
eV	-54.4	-13.6	-6.80	-6.05	-3.40

Table: Energy thresholds of $e^+e^-He^{++}$ binary channels

7 open channels between $He^+(n=3)$ and $He^+(n=4)$ thresholds.



Cross sections $e^+e^-He^{++}$

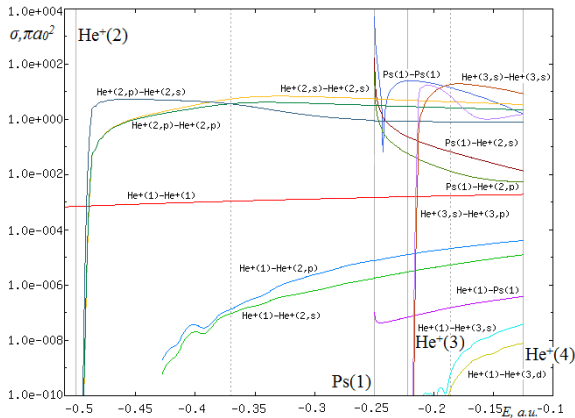


$e^+e^-He^{++}$ resonances

$(-0.3705, 0.1294)$	$(-0.250014, 7.4 \cdot 10^{-6})$	$(-0.1856, 0.0393)$
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Table: Known resonance energies (E_r, Γ) (in a.u.)

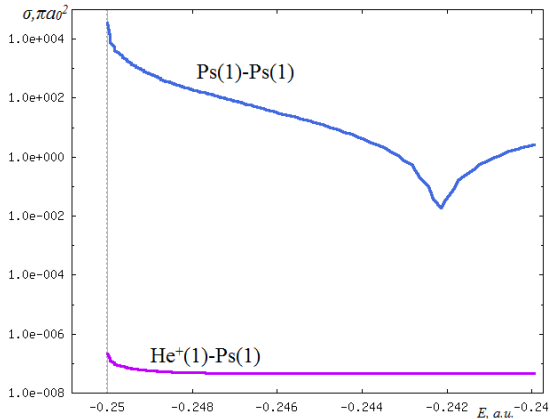
A. Igarashi and I. Shimamura. *Phys. Rev. A*, 70:012706, 2004



$e^+e^-He^{++}$ resonances

Process:	not charged	charged
elastic	const	$1/p^2$
slow→fast rearrangement	$1/p$	$1/p^2$
fast→slow rearrangement	p	const

$$p \equiv \sqrt{E - E_{th}}$$



$e^+e^-He^{++}$ resonances

Complex rotation method applied to the Schrödinger equation: broad resonances exist!

*		(-0.3704, 0.1297)		(-0.1857, 0.0395)
* *		(-0.3705, 0.1294)		(-0.1856, 0.0393)

Table: Broad resonance in the $e^+e^-He^{++}$ system energies (E_r, Γ) (in a.u.)

* Gradusov, Roudnev, Yarevsky, Yakovlev J. Phys. B: At. Mol. Opt. Phys. 52(2019) 055202

** A. Igarashi and I. Shimamura. *Phys. Rev. A*, 70:012706, 2004



VI. Conclusion

- The induced dipole interaction plays an important role in the three-body collision processes generating multiple resonances and specific oscillations of cross sections in Coulomb systems.
- Taking into account the contribution of the dipole interaction potential into the asymptotic boundary condition is decisive for correct treatment of the scattering problem in the three-body Coulomb systems.

Collaborants

This report is based on joint work with

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THANK YOU FOR YOUR ATTENTION

